Envisioning the Invisible

Tim Chartier

To Be the Remainder

"What is it like to be the remainder?" I answer this question with a mime sketch in a performance created specifically to motivate thinking about mathematics. I play the part of a clown who, almost compulsively, divides objects into two groups as they emerge from a suitcase. Repeatedly, one object is left: the remainder of the division. The pantomime touches on a universal human experience—being alone. In fact, the dramatic line of the piece relies heavily on a tension between togetherness and separateness.

This sketch guides the audience, mathematicians and nonmathematicians, through a cerebral journey of parity. The mathematical claim is quite simple: division by two of any odd number will result in a remainder of 1, whereas two divides evenly into any even number. The clown, in his simpleminded way, struggles to accept this fact. With increasingly large numbers, he is empathetically saddened by a sole remainder and satisfied when groups divide evenly. Yet, almost tempting mathematical fate, he peers into his suitcase even after forming groups of equal size. This daring spirit results in new, larger sets of objects that he must divide.

The sketch begins with one, then two, and quickly three objects. Even as the set grows to six and seven items, the audience can mentally group the objects to quickly discern the parity of the set. Later in the sketch, the set grows first by three and then by five. At this stage, it is harder to visually anticipate the resulting remainder. Primary school audiences are often heard counting the number of objects in order to calculate whether such aloneness will result. The clown almost always seems to discover his mathematical destiny more slowly than the audience. Children and adults alike often giggle when they compute sets of odd size.

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This sketch, like many others that I perform, contextualizes a mathematical idea, enabling an audience to place an abstract concept into a story. This mathematical mime piece presents mathematics in a physical rather than algebraic way. Whether written in formulas or portrayed through the movement of mime, such expressions are pointers to an intellectual experience. This sketch does not offer a proof of the divisibility of odd and even numbers. It does reflect what mathematicians often do prior to writing a proof—analyze simple examples and look for trends in cases of increasing complexity.

This sketch is quite popular with audiences of varying ages. Nonmathematicians enjoy anticipating the clown's ultimate fate with the set of ever-growing size. Mathematicians also enjoy the sketch. The concept it portrays is quite elementary. Mathematicians can be silenced by elegant proofs of simpler concepts and somehow dissatisfied with seemingly clumsy proofs of complex material. The pleasure brought about by a good proof can similarly be evoked through the performing arts—in this case, through a mimetic translation of mathematical thoughts. Why shouldn't we be silenced by an elegant sketch? Completed proofs uncover some truth; we have all experienced the exhilaration of such discoveries. Mathematicians can be quite excited by the mime sketches I perform as they illustrate their abstract world. Regarding this sketch on remainder, a child from a multi-age third- and fourth-grade classroom wrote,

> Thank you for coming to my class to teach and show us some pantomime…. I especially loved the remainder act because it made me laugh. I DID see the remainder.

Figure 1. Tim Chartier cuts a mime rope of infinite length.

It is natural to ask, "What mathematical ideas can be visualized with mime?" For some, math and mime might seem to exist in disjoint subspaces of the human experience, and thus a list of answers to this question would be an empty set. As for me, mime and math have converged in my life, and as reflected in this article, I see that the set is nonempty and countable. Yet, I sometimes wonder if the set is countably infinite or quite possibly of a finite size of such magnitude that I will never create a one-to-one correspondence between my mime sketches and invisible concepts of math that could be made visible with mime. In this article, I describe some of the ways I use mime to speak about mathematics.

Touching the Infinite

Let's now turn our attention to a mime piece that visualizes the one-dimensional number line as a rope of infinite length. The sketch begins with the lone mime walking toward the audience and suddenly stumbling. Peering down, my character sees an (invisible) object on the floor and proceeds to slowly pick it up. Examining it, he discovers a rope of infinite length in both directions. Wondering what might be at the end of this very long rope, he begins pulling and suddenly is pulled. A tug-of-war results and can prompt questions for the audience about the nature of infinity. My character becomes so frustrated that he pulls a pair of scissors from his back pocket and cuts the rope. Two pieces of the rope remain; one is anchored in his hand while the other portion zips away as if retracted far offstage. The plot continues, but I will refrain from divulging all of its secrets.

It is helpful to note that I, like many mimes, do not wear whiteface. Such a choice enables me to vary the presentation by utilizing masks and props and transition from silence to speaking. Given this, the piece easily provokes discussion on infinity. The portion of the sketch just described leads to the following question for the class or audience:

> There is the moment in the sketch when the mime cuts the rope that goes on forever in both directions. After it is cut, how long is the rope that remains in the mime's hand?

Audiences vary in their responses. My favorite answers generally come from children, which have included, "very long", "half as long", and, of course, "infinitely long". I generally state that the concept of infinite size differs from finite size in that something of infinite size can have some subset (itself possibly infinite in size) removed and remain of infinite size. Audience members have commented after a show, "I had always just thought of infinity as essentially the largest number." The mime sketch challenges this misconception.

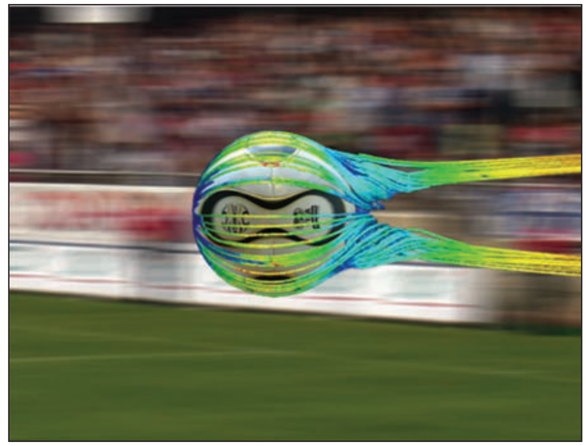
Mime speaks in its silence, often leaving echoing, unspoken words in the audience's consciousness. The contents of a clown's suitcase demonstrate a sense of number. An invisible rope left on the performance floor pulls one into struggles regarding cardinality and the infinite.

Math with a Mime Ball

Now, we will touch several arithmetic operators by, first, snatching a mime ball from the air. It has the weight of an orange. We should practice tossing and catching it several times. Of course, we are handling a mime ball, so we can just as easily pick another. Let's do so, and then smash it together with the first and reform the resulting mass into a mime ball with twice the weight as the original. We toss the newly formed ball into the air and catch it with appropriate adjustments for the new weight. Putting this ball aside, we snatch five mime balls from the air, mash them together, and demonstrate the resulting weight with physical changes in our tossing. Now, we smash 100 mime balls into one; the ball has grown so heavy that we cannot lift it. So we reverse the process and divide the ball in half and reform the hemisphere into a mime ball. We can repeat this process, as desired, until we have again produced an invisible object of liftable weight.

The extent to which the illusion of tossing a mime ball looks real depends largely on the ability of the mime to accurately approximate a realistic trajectory of the ball. The mime's body should reflect the force needed to throw a heavier object. The initial velocity of the ball should influence the length and height of the trajectory. Although a mime has the ability to create wildly unrealistic

(a)



(b)

Figure 2. (a) Children hold huge mime balls. (b) This illusion is connected to research in computational fluid dynamics to predict the flight of a soccer ball.

trajectories, an effect that can be quite humorous, such decisions are generally most effective after establishing the illusion of realistic movement.

At this point, audiences are inspired to learn about applications in science that compute trajectories. Predicting an object's trajectory under a set of initial conditions is a question appropriate for mathematical research. For instance, research by the University of Sheffield and Fluent Europe digitized a soccer ball and computed the resulting air flow over a kicked ball. (See Figure 2b.) Researchers found that the shape and surface of the ball, as well as its initial orientation, play a fundamental role in the ball's trajectory through the air. Such research could influence the stitching patterns of future soccer balls [3]. Note, however, that the goal of this scientific research is to accurately predict movement, whereas the goal of our mime movement was to simply seem believable.

Tubular Topology

In a piece with my wife and fellow trained mime, Tanya Chartier, her character interacts with a huge tube, as seen in Figure 3. Throughout the sketch, the tube, or "Slinky" as many audience members name it, contorts into a variety of shapes from small to tall, linear to twisted. The progression of shapes is actually choreographed so as to disorient many viewers as to my orientation. A popular discussion after the sketch involves audience members postulating how my body was positioned in the tube so as to construct different shapes.

Audiences seem ready to tackle mathematical topics that involve the tube. So we present a game. Suppose Slinky begins in the shape of a "$\vee$". Now, Slinky cannot disconnect any part of itself or attach itself to any other part at any time. Following only this simple rule, what letters of the alphabet can Slinky form? Quickly, audiences, from schoolchildren to residents of a retirement community, call out their answers. Note that the validity of one's answer depends, in part, on the style of font that one imagines. Whether this subtlety is mentioned depends largely on the mathematical sophistication of the audience.

This sketch places topology into a fun, nonthreatening setting. Audiences think, risk, and postulate. Isn't mathematics, from a certain viewpoint,

(a)

(b)

Figure 3. A huge Slinky motivates discussion on topology and equivalence of shapes.

Figure 4. Performances by Karl Schaffer (left) and Art Benjamin (right) focus on mathematical topics.

Photo on left by Steve DeBartolomeo and on the right by Anne White. Used with permission.

a cognitive game in which we ramble through our thoughts and options in search of a winning insight? When we "win", we celebrate, even if only momentarily. However, in this game, there is always another challenge, and successes generally lead to greater challenges and bigger rewards.

Being Isaac Newton

Ultimately, audiences, whether in primary, middle, or high schools, large state universities or private colleges, can live mathematics vicariously through my mime performance. In fact, a developing sketch will allow audience members to "meet" a mathematician like Isaac Newton through the use of mask work. I have studied under Marcel Marceau, and I have also trained in other schools of mime and physical theater, resulting in my broad and versatile approach to the art. These sketches on famous mathematicians will use masks constructed from a cast of my face and enable me to speak. Indeed, the sketch combines mime and spoken word. In time, I hope to develop robust sketches to match some tremendous masks, which I commissioned with grant funding, of Isaac Newton, Pierre de Fermat, and Sophie Germain. In performance, the characters will share stories from their lives, which ultimately led each to far-reaching impacts in mathematics. I select stories that reflect the humanity of the person and that may resonate with an audience member's life. Sophie Germain's mask is modeled from pictures of her as a youth. She will enter the stage with a blanket draped over her shoulders, as her parents hid her clothes in the evening in the hope that the cold night air would confine her to bed and stop her studying. Secretly, she will be stepping to her desk to study her beloved mathematics in the dark hours of the night. Pierre de Fermat will tell the tale of that fateful day in which a proof appeared to unfold in his mind but was too long to fit into the margins of his book. He could only write the conjecture, leaving centuries of mathematicians with the challenge of proving it true. Mathematical discovery is a human achievement. Such stories can remind audiences of the personal side and challenges that are all too often hidden behind the mathematical theorems and methods we study and develop.

One Act Among Many

My mathematical mime leans heavily on that performing art's ability to embody the invisible. Other mathematicians with a talent for other performing arts present mathematics in their own interesting and compelling ways [5].

Let me assemble a playbill of performing arts that could easily fill a mathematical variety show. We could begin with Colm Mulcahy (Spelman College) performing magic with a deck of cards. Knowing the mathematical sophistication of this audience, his tricks would lean on a variety of topics from Fibonacci numbers [6] to results of Paul Erdős [7]. After this opening act of magic, Colin Adams (Williams College) could enter as Mel Slugbate, who pitches investments in hyperbolic space for those nervous about the financial risks of buying realty in Euclidean space. Then, an audience member could select a circuit in a graph that would determine a sequence of 5-ball juggling patterns for Greg Warrington (University of Vermont). The lights dim and Karl Schaffer (DeAnza College) and a member of his troupe would begin a dance piece about the prisoner's dilemma. Finally, Art Benjamin (Harvey Mudd College) would enter and befuddle the audience with his outstanding rapid mental arithmetic and mathemagic. Interested in such an evening of entertaining mathematics? While I don't have a venue for such performances, I can lead you to a collection of work that you can read or view. In particular, see [1, 2, 4, 8, 9].

Silent, Closing Confessions

At a recent show in a rural middle school, the throng of early teenagers entered the auditorium with varying levels of interest; some cracked jokes about mimes and others spoke loudly regarding their frustrations with math. As I looked out on the assembled mass, a large proportion of them sat, in various poses, with that characteristic air of distant disdain of a teenage student. In performance, I generally make a quick judgment call as to how to begin the show. In this case, some well-placed jokes and the use of several convincing mime illusions transformed the atmosphere from quiet and uncertain to laughing and engaged. The show ended in that unmistakable, energetic applause of appreciation.

As I conclude this article and written presentation of my mathematical mime, I wonder what type of audience member you are. What auditorium did I enter as I began this article with the *Notices* readership? More important, have you touched the unseen or heard the unspoken through this narrative of my silent movement?

As mathematicians, our intellectual world is commonly abstract and invisible. We create a narrative of our intellectual thought through our written words. For me, mime is another, quite different but surprisingly similar, way of journeying through this logical field. Best of all, this art creates a road map that invites mathematician and nonmathematician alike to travel alongside me—often in silence, with occasional laughter.

Acknowledgments

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